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THE MOTION OF A PLANE EVAPORATION FRONT IN A SUPERHEATED LIQUID

by

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ABSTRACT

The velocity of the interface between liquid and vapor phases when a slightly superheated liquid changes to vapor is found. The temperature distribution in the liquid ahead of the evaporation front can then be determined by a numerical integration.

NOMENCLATURE

A	=	length scale factor defined in equation (20)
b	=	slope, dp/dt, of the saturation curve
c	=	speed of sound in the liquid
$g(t)$	=	position (i.e. x co-ordinate) of interface at time t
$g'(t)$	=	velocity of interface
k	=	thermal conductivity
L	=	latent heat of evaporation
m	=	mass rate of evaporation per unit area
p	=	pressure
t	=	time
T	=	temperature
u	=	fluid velocity
x	=	space co-ordinate
y	=	$x-g(t)$ = space co-ordinate measured from interface
α	=	thermal diffusivity
η	=	y/A = dimensionless distance from interface
θ	=	dimensionless temperature defined in equation (9)
ρ	=	density of liquid
ρ_v	=	density of vapor
τ	=	$\alpha t/A^2$ = dimensionless time
ϕ	=	dimensionless velocity of interface defined in equation (14)

SUBSCRIPTS:

o, refers to conditions at interface, liquid side

s, refers to equilibrium saturation condition

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INTRODUCTION

This study is an attempt to solve the problem of the motion of a plane front in a fluid medium across which a change of phase takes place. In particular, we assume that an infinitely long tube is filled with a liquid at a uniform pressure, $p(\infty)$, and at a uniform temperature, $T(\infty)$, slightly above the saturation temperature corresponding to $p(\infty)$. The liquid is thus in an unstable state.

We assume that at some point in the tube ($x=0$) some vapor is formed, and that we then have an evaporation front moving into the superheated liquid. As fluid crosses this front it is converted into a vapor at equilibrium saturation conditions. The energy to supply the latent heat of evaporation is assumed to come by conduction through the liquid. Hence the liquid will be cooler at the interface than at infinity.

We let $x = g(t)$ be the location of the front and $g'(t)$ its velocity. The determination of this function, $g'(t)$, is the principal object of the investigation.

ANALYSIS

The temperature in the liquid must satisfy the one-dimensional diffusion equation in a moving substance,

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial x^2} \tag{1}$$

in the region $x \geq g(t)$, $t \geq 0$, subject to the boundary conditions:

$$\left.\frac{\partial T}{\partial x}\right]_{x=g(t)} = \frac{L}{k}\,m(t) \tag{2}$$

$$T(x,0) = T(\infty) \tag{3}$$

$$T(\infty,t) = T(\infty) \tag{4}$$

In boundary condition (2), which expresses the fact that the latent heat of evaporation is supplied by conduction through the liquid, the functions g(t) and m(t) are unknown. If we assume that the moving front converts the liquid into stationary vapor, then

$$m(t) = \rho_v\, g'(t) = \rho\left[g'(t) - u_o\right] \tag{5}$$

from which

$$u_o = \left(1 - \frac{\rho_v}{\rho}\right) g'(t) \tag{6}$$

At 212^{o}F, $\frac{\rho_v}{\rho} = 7.5 \times 10^{-4}$; hence it is permissible in considering the effect of motion of the liquid to take

$$u_o = g'(t) \tag{7}$$

The curved boundary x = g(t) can be removed by choosing a co-ordinate measured from the interface. That is, letting y = x-g(t), equation (1) becomes

$$\frac{\partial T}{\partial t} + \left[u - g'(t)\right]\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The temperature gradient will be large only near the boundary $y = 0$, and here $[u - g'(t)]$ is small because of (7). Hence the convective term in (8) is neglected.

The problem is reformulated in dimensionless quantities by writing

$$\eta = \frac{y}{A}\ , \qquad \tau = \frac{\alpha t}{A^2}\ , \qquad \theta = \frac{T - T(\infty)}{T(\infty) - T_s(\infty)} \tag{9}$$

where $T_s(\infty)$ is the equilibrium saturation temperature corresponding to $p(\infty)$. We then have the standard diffusion equation

$$\theta_\tau = \theta_{\eta\eta} \tag{10}$$

(the subscripts denote partial differentiation) to be solved in the region $\eta \geq 0$, $\tau \geq 0$, with the boundary conditions:

$$\theta(\eta, 0) = 0 \tag{11}$$

$$\theta(\infty, \tau) = 0 \tag{12}$$

$$\theta_\eta(0, \tau) = \phi(\tau) \tag{13}$$

The boundary function $\phi(\tau)$ in (13), still to be determined, is the dimensionless form of the velocity of the interface, $g'(t)$, as is seen from (2), (5), and (9).

$$\phi(\tau) = \frac{A L \rho_v\, g'(t)}{K[T(\infty) - T_s(\infty)]} \tag{14}$$

The boundary value problem, equations (10), (11), (12), (13), is most conveniently solved by applying the Fourier cosine transform on the variable η. Let the transform be denoted by

$$\bar{\theta}(\lambda, \tau) = \sqrt{\frac{2}{\pi}} \int_0^\infty \theta(\eta, \tau) \cos \lambda\eta \ d\eta$$

For the boundary conditions of the present problem the partial differential equation (10) transforms to

$$\frac{d\bar{\theta}}{d\tau} = -\frac{2}{\sqrt{\pi}}\,\phi(\tau) - \lambda^2\bar{\theta}$$

with the solution

$$\bar{\theta} = -\frac{2}{\sqrt{\pi}}\int_0^{\tau} e^{-\lambda^2(\tau - s)}\,\phi(s)\,ds \tag{15}$$

It can be shown that the inversion of (15) is

$$\theta(\eta,\tau) = -\frac{1}{\sqrt{\pi}}\int_0^{\tau} e^{-\frac{\eta^2}{4(\tau-s)}}\,\phi(s)(\tau-s)^{-\frac{1}{2}}\,ds \tag{16}$$

giving the temperature distribution in terms of the motion of the interface.

From (16) we find the temperature at the interface to be

$$\theta_0 = \lim_{\eta\to 0}\theta(\eta,\tau) = -\frac{1}{\sqrt{\pi}}\int_0^{\tau}\frac{\phi(s)}{\sqrt{\tau - s}}\,ds \tag{17}$$

Since thermodynamic equilibrium is established at the evaporation front, the boundary temperature, $T_0(\tau) = T(0,\tau)$, given by (17), will determine the pressure at the interface $p_0(\tau)$. If the range of temperature variation is small enough, we can take the relationship between pressure and temperature at saturation to be linear. That is,

$$\begin{aligned} p_0 &= p(\infty) + b\left[T_0 - T_s(\infty)\right] \\ &= p(\infty) + b\left[T(\infty) - T_s(\infty)\right]\left[1 - \frac{1}{\sqrt{\pi}}\int_0^{\tau}\frac{\phi(s)}{\sqrt{\tau-s}}\,ds\right] \end{aligned} \tag{18}$$

We now assume that the evaporation front moving into the originally motionless liquid sends a wave into this liquid in which the fluid velocity and the pressure are related as in an acoustic medium.

$$p(x,t) = p(\infty) + \rho c u(x,t)$$

In particular, at $x = g(t)$

$$\begin{aligned} p_0 &= p(\infty) + \rho c\, g'(t) \\ &= p(\infty) + \frac{\rho c\, K\left[T(\infty) - T_s(\infty)\right]\phi(\tau)}{\rho_v L A} \end{aligned} \tag{19}$$

If we now choose our length scale A to be

$$A = \frac{\rho c K}{\rho_v L b} \qquad (20)$$

equating (18) and (19) provides the following integral equation for the motion of the interface:

$$\phi(\tau) = 1 - \frac{1}{\sqrt{\pi}} \int_0^{\tau} \frac{\phi(s)}{\sqrt{\tau - s}}\, ds \qquad (21)$$

Equation (21) is solved by an iterative technique in which

$$\phi_{n+1}(\tau) = 1 - \frac{1}{\sqrt{\pi}} \int_0^{\tau} \frac{\phi_n(s)}{\sqrt{\tau - s}}\, ds$$

with $\phi_1 = 1$, the iteration can be carried to any order with only elementary integrals being encountered. The result is the following power series:

$$\phi = \left(1 + \tau + \frac{1}{2}\tau^2 + \frac{1}{6}\tau^3 + \cdots\cdots\right) - \frac{2}{\sqrt{\pi}}\left(\tau^{\frac{1}{2}} + \frac{2}{3}\tau^{\frac{3}{2}} + \frac{4}{15}\tau^{\frac{5}{2}} + \cdots\cdots\right)$$

After some manipulation the series can be recognized as the expansion of

$$\phi = e^{\tau} \cdot \operatorname{erfc}\left(\tau^{\frac{1}{2}}\right) \qquad (22)$$

where erfc denotes the complimentary error function defined by

$$\operatorname{erfc}(z) = 1 - \operatorname{erf}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{z} e^{-v^2}\, dv$$

RESULTS

Fig. 1 is a graph of equation (22). Fig. 2 shows some typical temperature distributions obtained by numerical evaluation of the integral in equation (16).

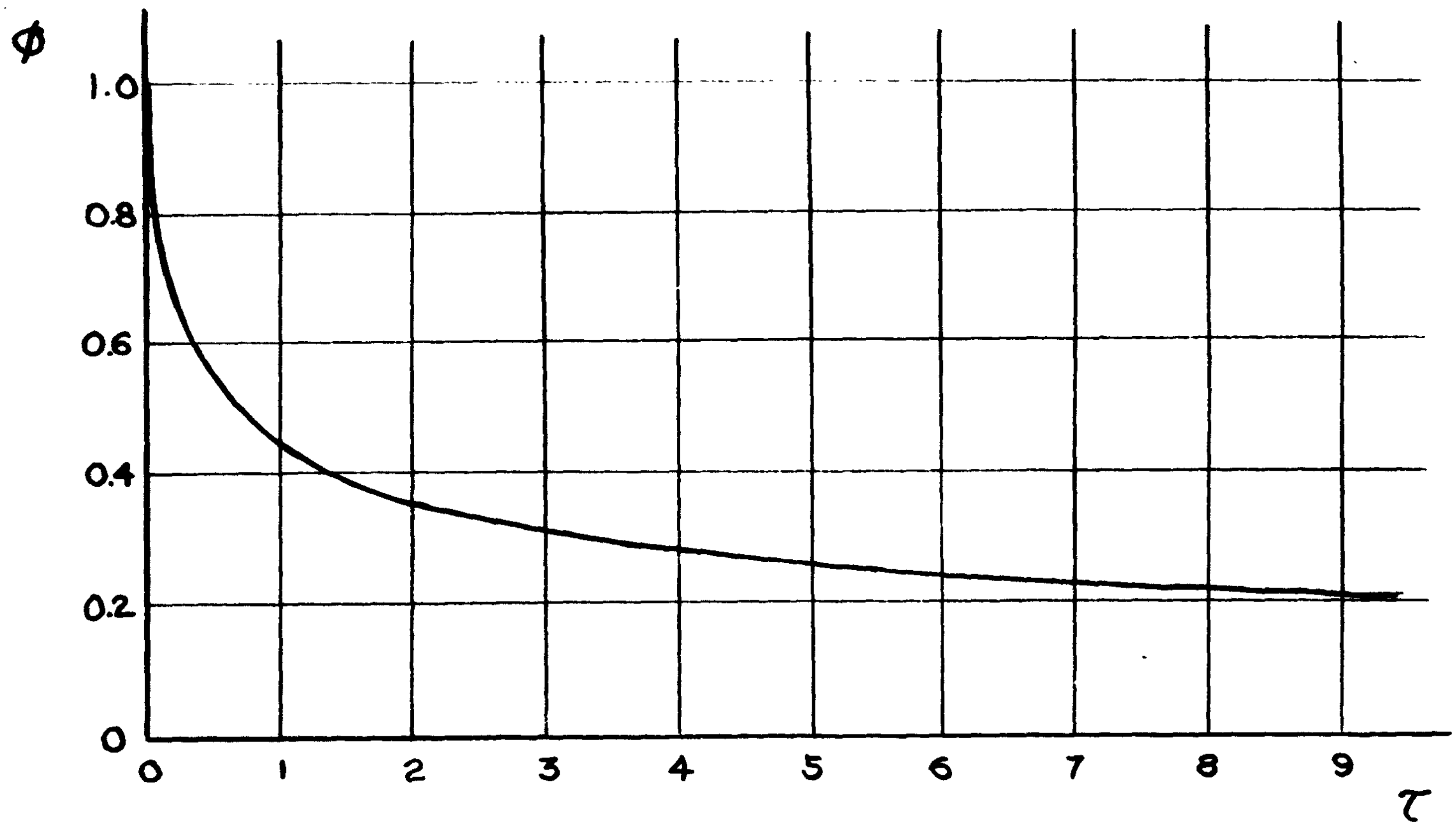

FIG. 1 VELOCITY OF INTERFACE VS. TIME

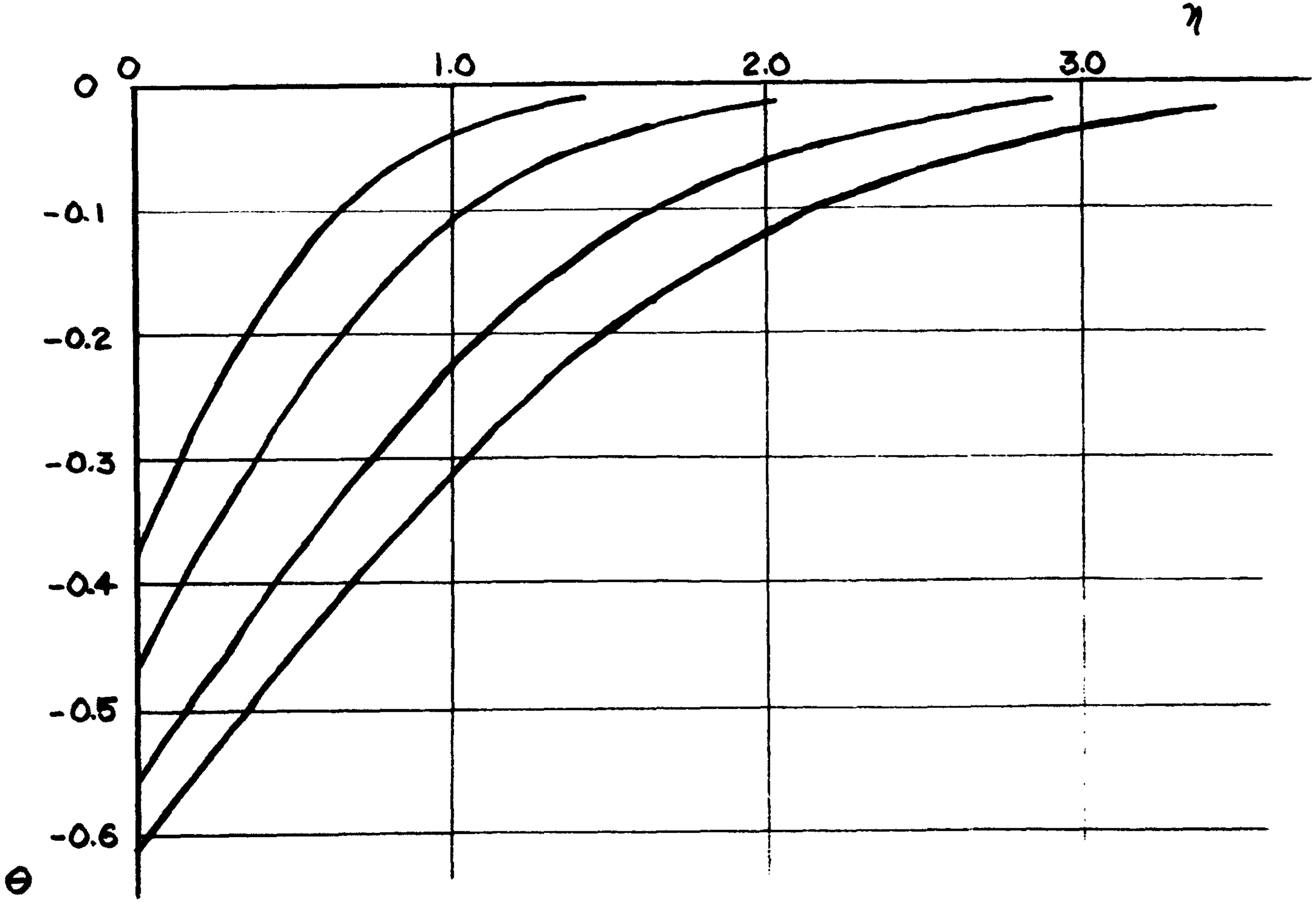

FIG. 2 TEMPERATURE DISTRIBUTION

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